Non-Acceleration Theorem in a Primitive Equation System: I. Acceleration of Zonal Mean Flow[1]

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ABSTRACT

   Non-acceleration theorem in a primitive equation system is developed to investigate the influences of waves on the mean flow variation against external forcing. Numerical results show that mechanical forcing overwhelms thermal forcing in maintaining the mean flow in which the internal mechanical forcing associated with horizontal eddy flux of momentum plays the most important roles. Both internal forcing and external forcing are shown to be active and at the first place for the mean flow variations, whereas the forcing-induced mean meridional circulation is passive and secondary. It is also shown that the effects on mean flow of external mechanical forcing are concentrated in the lower troposphere, whereas those due to wave-mean flow interaction are more important in the upper troposphere. These act together and result in the vertically easterly shear in low latitudes and westerly shear in mid-latitudes. This vertical shear of mean flow is to some extent weakened by thermal forcing.

I. INTRODUCTION

   As early as the theory of the mean meridional circulation (MMC) of the atmosphere was established in 18th century, meteorologists realized the connections between the mean meridional circulation and mean zonal surface winds (Lorenz, 1967). One of the problems turned out to be critical in atmospheric dynamics then was how the mean flow of atmospheric motions can be maintained, against surface frictional dissipations. The earliest satisfactory interpretation on this was due to Jeffreys (1926) who, in a study on the dynamics of geostrophic winds, postulated that cross-latitude transfer of momentum should be responsible. This postulation obtained strong supports and became evident in the late 1940's and early 1950's as a result of series of diagnostic studies of Starr (1948), Widger (1949) and White (1949) and so forth. However, it was not until the 1960's that the systematic theoretical investigations on the wave-mean flow interactions started. In their studies on the propagations of stationary wave and wave energy, Eliassen and Palm (1960), and Charney and Drazin (1961) found independently the relations between mean flow acceleration and wave activities. The EP-theory and non-acceleration theorem, which stress one problem in different ways, were then put forward. Since then innumerable investigations have contributed to the development of

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wave—mean flow interactions. In 1976, Andrews and McIntyre developed the transformed mean flow equation system, and defined the EP—flux and residual mean meridional circulation. In 1980, Edmon, Hoskins and McIntyre (refer to EHM hereafter) extended the quasi—geostrophic non—acceleration theorem to cover the interactions of mean flow with eddies of any amplitudes, developed the generalized EP—flux and compared the interaction of mean flow with stationary waves to that with transient eddies. Stone and Salustri (1984) introduced the effects of eddy water flow to the subject, developed the moist EP—cross—section and proved the applicability of the non—acceleration theorem in a moist atmosphere. By decomposing the eddy transfer processes into different wave domains, Wu et al.(1988) found the significant differences between interactions of mean flow with planetary scale waves and synoptic scale waves in either dry or moist model.

Along with the development of stationary wave propagations, the transiency and temporal evolutions of both mean flow and eddies were systematically studied by Zeng (1979, 1983, 1985, 1986) based upon the employment of wave packet theory. Zeng pointed out that when the inertia axis of the atmosphere coincides with the rotating axis of the earth, unstable waves, no matter barotropic or baroclinic, should eventually be absorbed by the mean flow. Only when the inertia axes of atmospheric motions are independent of the earth-rotating axis, can the large—scale eddies be quasi—stationary. The wave packet theory of Zeng also shows that the interactions between waves and mean flow depend not only on the intensity and horizontal and vertical shears of the basic flow but also on the wave scale and structure.

Numerical analyses and modelling can also reveal the characteristics of wave—mean flow interactions. One of these investigations was taken by Pfeffer (1981). By virtue of the elegant model of Kuo (1956) he was able to compare the relative significance of eddy fluxes of heat and momentum with that of diabatic heating in the changes of atmospheric basic states. Due to the lack of global data, he had to use the northern hemispheric analyses of Oort and Rasmusson (1971) and to regard the equator as a seasonally symmetric axis in the $\varphi-p$ cross—section so that the southern summer (DJF) atmosphere could be a simple reflection of the northern summer (JJA) atmosphere and so forth. This is, of cause, not the case. In addition, in the statistics of Oort and Rasmusson, the "transient flux" contains not only the transient eddy flux (e.g., $[v^{*'} A^{*'}]$) , but also the transient flux of the zonal—mean flow (e.g., $[v] [A]'$. By this reason, it is not exactly suitable to use the data set to study the wave—mean flow interactions in so far as the transiency of eddies is involved. Despite all this, Pfeffer's work was excellent in that it tells us that when the wave—mean flow interaction is studied, the effects on the mean atmospheric states of the eddy—induced meridional circulation can not be ignored.

In this study, we will discuss the non—acceleration subject in primitive equation framework. Then, by employing the time—mean global statistics of Wu and Liu (1987), the variations of mean flow when it interacts with eddies will be compared to that due to external mechanical and thermal forcing.

II. NON—ACCELERATION THEOREM IN PRIMITIVE EQUATION SYSTEM

For the sake of comparison, before we discuss the non—acceleration problem in primitive equation system, the subject in quasi—geostrophic framework will be briefly reviewed.

1. Quasi—Geostrophic Non—Acceleration Theorem

It is convenient to start with the transformed mean flow equation system used in EHM. In the p—coordinate, the momentum equation, thermal wind relation, continuity and thermodynamic equation can be expressed as

$$[u]_t = f_0 \tilde{v} + (a\cos\varphi)^{-1} \nabla \cdot \vec{E} + \mathscr{X}$$ (1)

$$f_0 [u]_p - a^{-1} R^{\star} [\theta]_\varphi = 0$$ (2)

$$(a\cos\varphi)^{-1} (\tilde{v}\cos\varphi)_\varphi + \tilde{\omega}_p = 0$$ (3)

$$[\theta]_t + \Theta_p \tilde{\omega} = Q_m,$$ (4)

where $R^{\star} = \dfrac{R}{p}(\dfrac{p}{p_0})^\kappa$, $\mathscr{X}$ and Q_m are sources of momentum and heat respectively, EP—flux

$$\vec{E} = (E(\varphi), E(p)) = a\cos\varphi(-[u^* v^*], f_0 \Theta_p^{-1} [v^* \theta^*])$$ (5)

and residual circulation

$$\begin{cases} \tilde{v} = [v] - (\Theta_p^{-1} [v^* \theta^*])_p \\ \tilde{\omega} = [\omega] + (a\cos\varphi)^{-1} (\Theta_p^{-1} [v^* \theta^*]\cos\varphi)_\varphi \end{cases}$$ (6)

in which "[ ]" and " * " denote the zonal mean and deviation therefrom. From (3) and (6), the streamfunction for the residual circulation can be defined from

$$\begin{cases} 2\pi a g^{-1} \cos\varphi \tilde{\omega} = a^{-1} X_\varphi \\ 2\pi a g^{-1} \cos\varphi \tilde{v} = X_p \end{cases}$$ (7)

Let Θ_p be a constant, if $\mathscr{X} = Q_m = \nabla \cdot \vec{E} = 0$, then from (1) to (4) the following equation can be reached:

$$f_0^2 \tilde{v}_{pp} + a^{-2} R^{\star} [\Theta_p][(\cos\varphi)^{-1}(\tilde{v}\cos\varphi)_\varphi]_\varphi = 0,$$ (8)

then $\tilde{v}$ possesses trivial solution provided $\tilde{v}$ equals zero at the boundary. Thus, from (1) and (4)

$$[u]_t = [\theta]_t = 0.$$ (9)

Therefore, the non—acceleration theorem in the dry model can be stressed as

$$\text{when } \mathscr{X} = Q_m = \nabla \cdot \vec{E} = 0, \ [u]_t = [\theta]_t = 0.$$ (10)

This means that, when there are no external forcing $\mathscr{X}$ and Q_m, nor internal forcing $\nabla \cdot \vec{E}$, the mean state will be steady even though there exist waves.

In order to take the eddy water flux under consideration, Stone and Salustri (1984) introduced the following equation of water conservation

$$[q]_t + (q_s)_p[\omega] = -(C - S) - (a\cos\varphi)^{-1}[v^* q^* \cos\varphi]_\varphi, \tag{11}$$

where q is the specific humidity, and $q_s = q_s(\varphi, p)$, the time and zonal mean of q. Since at steady state, the convergence of eddy flux of water vapour $[-(q_s)_p\omega - (a\cos\varphi)^{-1}(v^* q^* \cos\varphi)_\varphi]$ must be balanced by either condensation C or other water sink $(-S)$, $(C-S)$ can then be regarded as condensation rate. Now, the heating rate Q_m in (4) can be decomposed into latent heating and other kinds of diabatic heating Q_d, i.e.

$$Q_m = Q_d + A(p)(C - S), \tag{12}$$

where $A(p) = LC_p^{-1}(p_0 / p)^\kappa$. Therefore, the transformed mean flow equation in the moist model can be written to

$$[u]_t - f_0\tilde{v}_m = (a\cos\varphi)^{-1}\nabla \cdot \vec{E}_m + \mathcal{I} \tag{13}$$

$$f_0[u]_p - a^{-1}R^* [\theta]_\varphi = 0 \tag{14}$$

$$(a\cos\varphi)^{-1}(\tilde{v}_m\cos\varphi)_\varphi + (\tilde{\omega}_m)_p = 0 \tag{15}$$

$$[\theta_c]_t + \Theta_{cp}\tilde{\omega}_m + \sigma^*[\omega] = Q_d \tag{16}$$

$$[q]_t + q_{sp}\tilde{\omega}_m = (S - C) + M \tag{17}$$

$$\theta_c = \theta + A(p)q \tag{18}$$

where, the subscript m stands for moist model,

$$\theta_{cp} = \Theta_{cp}(p) + \sigma^*(\varphi, p), \tag{19}$$

the function

$$M = -(a\cos\varphi)^{-1}[v^* q^* \cos\varphi]_\varphi + (a\cos\varphi\Theta_{cp})^{-1}q_{sp}[v^* \theta_c^* \cos\varphi]_\varphi \tag{20}$$

describes the eddy forcing of the moisture field, and the moist EP–flux and residual circulation are respectively defined by

$$\vec{E}_m = (E_m(\varphi), E_m(p)) = a\cos\varphi(-[u^* v^*], f_0\Theta_{cp}^{-1}[v^* \theta_c^*]) \tag{21}$$

$$\tilde{v}_m = [v] - [v^* \theta_c^* \Theta_{cp}^{-1}]_p \tag{22}$$

$$\tilde{\omega}_m = [\omega] + (a\cos\varphi)^{-1}[v^* \theta_c^* \cos\varphi\Theta_{cp}^{-1}]_\varphi. \tag{23}$$

In combination with (15), the streamfunction $\tilde{X}_m$ of the moist residual circulation can be defined from

$$\begin{cases} 2\pi a g^{-1}\cos\varphi\tilde{\omega}_m = -a^{-1}\tilde{X}_{m\varphi} \\ 2\pi a g^{-1}\cos\varphi\tilde{v}_m = \tilde{X}_{mp} \end{cases} \tag{24}$$

In extratropical area, $\sigma^*[\omega]$ is at least one order of magnitude smaller than $\Theta_{cp}\tilde{\omega}_m$, and can be omitted from (16). Therefore, on the analogy of the derivation in dry model, the non–acceleration theorem of the moist model can be stated as:

when $\qquad$ $\chi = Q_d = \nabla \cdot \vec{E}_m = 0$, and $M = C - S$,

then there exist trivial solution

$$\tilde{v}_m = \tilde{\omega}_m = [u]_t = [\theta]_t = [q]_t = 0. \tag{25}$$

Actually, it is readily to show that, in combination with (11), the system (1)–(4) is equivalent to the moist system (13)–(18). Both of them can be used to investigate the influences of eddy transfer on the mean flow. The only difference exists in that latent heating as one part of external forcing Q_m in the dry model has been transferred to an internal forcing associated with eddy transfer of water vapour in the moist model.

2. *Non–Acceleration Theorem in Primitive Equation System*

In quasi–geostrophic frameworks, the vertical transfer properties of eddies and of the MMC are all omitted. Data analyses showed that the convergences of these fluxes are by no means negligible in some cases in maintaining the atmospheric budgets. More importantly, the intensity of the Hadley cell is much too weak in quasi–geostrophic system, the application in low latitudes of the non–acceleration theorem in quasi–geostrophic frameworks is then limited. We now turn to seek the solution in primitive equation system. Firstly, we employ the method used by Kuo to derive the mean meridional circulation equation. The momentum equation, thermodynamic equation, geostrophic wind relation, continuity, hydrostatic equation and the definition of geopotential temperature can respectively be written to

$$[u]_t = [v](f - \frac{1}{a\cos\varphi}[u\cos\varphi]_\varphi) - [\omega][u]_p - \frac{1}{a\cos^2\varphi}[u^* v^* \cos^2\varphi]_\varphi - [u^* \omega^*]_p + \chi \tag{26}$$

$$[\theta]_t = -a^{-1}[v][\theta]_\varphi - [\omega][\theta]_p - \frac{1}{a\cos\varphi}[v^* \theta^* \cos\varphi]_\varphi - [\omega^* \theta^*]_p + Q \tag{27}$$

$$(f + a^{-1}[u]\tan\varphi)[u] = -a^{-1}[\Phi]_\varphi \tag{28}$$

$$(a\cos\varphi)^{-1}[v\cos\varphi]_\varphi + [\omega]_p = 0 \tag{29}$$

$$[\Phi]_p = -\frac{R}{p}[T] \tag{30}$$

$$\theta = T(\frac{p_0}{p})^\kappa. \tag{31}$$

Let

$$\begin{cases} A = -(a^2 \bar{\rho}\cos\varphi)^{-1}[\ln\theta]_p \\ B = (a^2 \bar{\rho}\cos\varphi)^{-1}[\ln\theta]_\varphi \\ C = (\cos\varphi)^{-1}f(f - (a\cos\varphi)^{-1}[u\cos\varphi]_\varphi) \end{cases} \tag{32}$$

$$\begin{cases} F_1 = -\dfrac{f}{a\cos^2\varphi}[u^*v^*\cos^2]_\varphi \\[2mm] F_2 = -f[u^*\omega^*]_p \\[2mm] F_3 = f\,\dot{r} \\[2mm] F = \sum_i F_i \qquad (i=\overline{1,3}) \end{cases} \qquad (33)$$

$$\begin{cases} H_1 = -\dfrac{R}{ap}(\dfrac{p}{p_0})^\kappa (a\cos\varphi)^{-1}[v^*\theta^*\cos\varphi]_\varphi \\[2mm] H_2 = -\dfrac{R}{ap}(\dfrac{p}{p_0})^\kappa[\omega^*\theta^*]_p \\[2mm] H_3 = \dfrac{R}{ap}(\dfrac{p}{p_0})^\kappa Q_m \\[2mm] H = \sum_i H_i \qquad (i=\overline{1,3}) \end{cases} \qquad (34)$$

and

$$\begin{cases} [v] = (\cos\varphi)^{-1}\psi_p \\[2mm] [\omega] = -(a\cos\varphi)^{-1}\psi_\varphi \end{cases} \qquad (35)$$

then, the momentum equation (26) and thermodynamic equation (27) can be written to

$$[u]_t = f^{-1}C\psi_p + [f+2a^{-1}u\tan\varphi]^{-1}B\psi_\varphi + f^{-1}(F_1+F_2+F_3) \qquad (36)$$

$$[\theta]_t = -(a\cos\varphi)^{-1}[\theta]_\varphi\psi_p + (a\cos\varphi)^{-1}[\theta]_p\psi_\varphi + \frac{ap}{R}(\frac{p_0}{p})^\kappa(H_1+H_2+H_3). \qquad (37)$$

Upon using the thermal wind relation

$$[f+2a^{-1}u\tan\varphi][u]_p = \frac{R}{ap}(\frac{p}{p_0})^\kappa[\theta]_\varphi, \qquad (38)$$

and noticing $|[u]|/(a\Omega\cos\varphi)\ll 1$, from (36) and (37) the mean meridional circulation equation (ψ—equation) can be obtained as

$$\psi - : \quad (A\psi_\varphi)_\varphi + 2B(\psi_{\varphi p}) + (C\psi_p)_p + B_p\psi_\varphi + B_\omega\psi_p$$
$$= (F_1+F_2+F_3)_p + (H_1+H_2+H_3)_\varphi = F_p + H_\omega. \qquad (39)$$

If A and B are taken to be independent of φ, the ψ—equation then turns out to be the same as equation (9) of Kuo. The general parameters of the atmosphere yields

$$B^2 - AC < 0. \qquad (40)$$

Thus, the ψ—equation is of elliptic type. For smooth undernearth, the boundary values of ψ can be taken as zero. Since for such type of equation, ψ can not possess its extremes at any inner point, when $F_i = H_i = 0$, $(i=\overline{1,3})$, we then have the solution

$$\psi \equiv 0. \qquad (41)$$

In such circumstances, $[u]_t = [\theta]_t = 0$. Therefore, if we denote

$$\nabla\cdot[\vec{v}u^*] = \frac{1}{a\cos^2\varphi}[v^*u^*\cos^2\varphi]_\varphi + [\omega^*u^*]_p = f^{-1}(F_1+F_2) \qquad (42)$$

$$\nabla \cdot [\bar{v}\theta^*] = \frac{1}{a\cos\varphi}[v^*\theta^*\cos\varphi]_\varphi + [\omega^*\theta^*]_p = \frac{ap}{R}(\frac{p_0}{p})^\kappa (H_1 + H_2), \tag{43}$$

then we obtain the non–acceleration theorem in the primitive equation system as follows:

If $\dot{\gamma} = Q_m = 0$, and $\nabla' \cdot [\bar{v}u^*] = \nabla \cdot [\bar{v}\theta^*] = 0$, then $[u]_t = [\theta]_t = 0$.   (44)

This means that for an isolated conservative system, when the zonal mean eddy fluxes of momentum and heat are of non–divergence, the zonal mean westerlies and temperature will not change with time. Notice that when (44) was derived, no assumption was made. Besides, in comparison with the non–acceleration theorem of filter model, not only the horizontal eddy fluxes, but also the vertical eddy fluxes are considered. Thus the non–acceleration theorem in the primitive equation system, i.e. (44), is more practical.

The atmosphere is a forced dissipative open system, the precondition $\dot{\gamma} = Q_m (= Q_d)$ = 0 in the non–acceleration theorem in either quasi–geostrophic system or primitive equation system is too restricted. Actually, it is apparent from (36) to (39) that as far as the external and internal forcing of angular momentum and heating are respectively in balance, i.e. the internal and external forcing cancel out one another rather than zero for the separate forcing, the non–acceleration theorem still holds. Therefore, the generalized non–acceleration theorem can be stated to:

If $F = 0$, $H = 0$, then $[u]_t = [\theta]_t = 0$.   (45)

This generalized theorem shows that, when and only when the internal and external forcing are not in balance, the mean meridional circulation can be excited, and the variations in basic flow and temperature can occur. In other words, the active and first causes of the temporal variation of mean westerlies are the internal forcing due to eddies and external forcing due to the existences of momentum and heat sources, and the passive and secondary cause is the effect of mean meridional circulation induced by both (either) internal and (or) external forcing.

III. THE EFFECTS ON MEAN FLOW OF INTERNAL AND EXTERNAL MOMENTUM FORCING

Let
$$\begin{cases} u_0 = f^{-1}C\psi_p + [f + 2a^{-1}u\tan\varphi]^{-1}B\psi_\varphi \\ u_E = f^{-1}(F_1 + F_2) \\ u_s = f^{-1}F_3 \end{cases} \tag{46}$$

then equation (36) can be written to

$$[u]_t = u_0 + u_E + u_s. \tag{47}$$

The three terms on the right hand side of (47) represent respectively the influences on the temporal variation of mean flow of forcing–induced mean meridional circulation, internal forcing due to eddy transfer properties, and external forcing due to momentum generation. Whenever the internal forcing u_E or external forcing u_S is given, the impact of the forcing–induced mean meridional circulation u_0 can be obtained via (39) and (46), and the temporal variation of $[u]$ can then be calculated from (47). By means of this, the wave–mean flow interaction can be investigated against the influences of momentum source on the mean flow. The time–mean global general circulation statistics of Wu and Liu (1987) offers a set of

data with complete separation of quantities between eddies and mean flow, and is suitable for the study of present purposes. The statistics is based on the ECMWF analysed data of four times per day ranging from September. 1979 to August 1984, with thirteen layers from 30 to 1000 hPa and horizontal resolution of 3 degrees of latitude from 87° S to 87° N. Heat and momentum sources were calculated as residuals of the corresponding governing equations with their local variations neglected,for their monthly means of January are really small.

Fig.1 shows the westerly accelerations in January due to the convergence of horizontal eddy momentum flux alone, i.e. $F_1 \neq 0$, $F_2 = F_3 = H_i = 0$ ($i = \overline{1,3}$). The direct effects of this internal forcing, i.e. u_E are mainly above 700 hPa (Fig.1a) and upward increasing with maxima near the tropopause. This forcing accelerates the mean westerly in mid–latitudes and decelerates the westerlies in high and low latitudes. The acceleration centres are located near 35° N and 45° S, with intensities of 4.3 and 5.2 $ms^{-1}d^{-1}$ respectively. The deceleration centres are located on the equatorward side of the jet centre in each hemisphere, with intensities of about 2 $ms^{-1}d^{-1}$ in the Southern Hemisphere and 3 $ms^{-1}d^{-1}$ in the Northern Hemisphere.

The indirect effects of forcing F_1 on mean flow, i.e. u_0 are shown in Fig.1b. Above 500 hPa, these effects are almost everywhere opposite to, and with intensities of about one third to one fourth of, those of the direct effects shown in Fig.1a. Below 500 hPa, the sign of u_0 is opposite to that above. Therefore, the F_1–induced mean meridional circulation is to decrease the westerly shear in mid–latitudes and to increase the shear in high and low latitudes resulting from horizontal eddy transfer of momentum.

The total momentum tendency due to F_1 forcing shown in Fig.1c possesses a distribution very similar to that of the direct effects of the forcing shown in Fig.1a. However, there exist two remarkable differences between them. Firstly, the intensities of maximum tendency $[u]_t$ are about one third weaker than those in u_E. Secondly the vertical westerly shear in $[u]$ is weakened compared to that in u_E. The effects of the secondary circulation on the intensity and structure of the zonal mean flow acceleration are evidently important.

The effects of eddies on the mean flow resulting from the vertical eddy transfer of momentum, i.e. F_2, as shown in Fig.2, are much concentrated to the upper troposphere. At first glance, either u_E or u_0 is quite strong with the magnitude of 10^{-5} ms^{-2}. However, they possess the opposite signs almost everywhere and cancel substantially out one another, so that the magnitude of $[u]_t$ due to F_2 is at least one order smaller than that due to F_1. Therefore, quasi–geostrophic system in which F_2 is omitted is valid as a first approximation in so far as the scale analysis is concerned.

If the internal forcing F_1 and F_2 are put together, the results of u_E, u_0 and $[u]_t$ (Fig.3) are very much similar to their counterparts in the F_1–forcing case shown in Fig.1. This hints that, when the primitive momentum equation is simplified, if the term $[\omega' u^*]_p$ is to be omitted, then, in order to keep the momentum balance, the part of momentum flux of the mean meridional circulation induced by $[\omega' u^*]_p$ must be taken out. Otherwise, as evident in Fig.2b, remarkable errors would occur.

Despite the difference in logics, the starting equations (26) and (27) and the methods of solving these equations are basically the same as those employed by Pfeffer (1981). Therefore, it is worthwhile to compare Fig.3 with the corresponding results shown in Fig.8 of Pfeffer

(1981). In the Northern Hemisphere, their corresponding distributions in u_E, u_0 and $[u]_t$ are quite similar, however, the intensities of the centres in u_E and $[u]_t$ in mid–latitudes are respectively 6×10^{-5} and 3×10^{-5} ms^{-2} in our results; but only 4×10^{-5} and 2×10^{-5} ms^{-5} in Pfeffer's calculations, about one third smaller. This might be due to interannual variation of the climate system. More importantly, as mentioned in the last section, data of F_1 employed in Pfeffer's calculation include the part $\overline{[\omega]'[u]'}$ which is in fact associated with the F_1–induced mean meridional flux of momentum, and contained in u_0. In other words, the term of u_E in Pfeffer's calculations already contains some part of u_0 in equation (47). As shown in Fig.1, u_0 might cancel out one third of u_E .Therefore, when the data set of Oort and Rasmusson (1971) is used to study the subject of wave–mean flow interactions, the influences of waves on mean flow should be weaker.

It should also be pointed out that in Pfeffer's calculations, although the dominant "eddy terms" correspond to our u_E term shown in Fig.3a, his $(\overline{v},\overline{\omega})$ terms were, however, forced by both eddy flux of momentum and heat $(F_1 + H_1)$, corresponding to the combination of Figs.1b and 6a in our present study, and therefore not the same as our u_0 term shown in Fig.3b. This accounts for the appearance of the mid–latitude surface acceleration centres in Pfeffer's results.

The distributions of u_S, u_0 and $[u]_t$ due to external mechanical forcing (F_3) are shown in Fig.4. Above 500 hPa, the direct effects on mean flow of external forcing shown in Fig.4a are opposite to those of internal mechanical forcing shown in Fig.3a. The deceleration of the mid–latitude westerlies due to external forcing F_3 cancels about one third of the acceleration due to internal forcing. The main effects of external forcing are found below 800 hPa. In connection with mountain stress and frictional stress, westerly momentum of the atmosphere is substantially generated in low latitudes between $35°$ S and $35°$ N, and dissipated in mid–latitudes. However, this forcing effect is greatly cancelled by the forcing–induced meridional circulation as shown in Fig.4b. The total effects of external forcing F_3 shown in Fig.4c are to decelerate in mid–latitudes and accelerate in high and low latitudes the westerlies of the atmosphere in a more uniform way in the vertical. The low layer data in southern high latitudes are unreliable. Except this part, the intensity of $[u]_t$ due to external forcing is in the order of $1.5–3$ ms^{-1}day^{-1}. In mid–latitudes, this is about half at the tropopause, and four times at the surface, as large as that due to total eddy forcing (F_1+F_2) (Fig.3c).

If the total internal and external forcing $F_i (i = 1,3)$ act together, their impacts on the temporal variation of westerly momentum are shown in Fig.5. Again, the direct forcing effects $(u_E + u_S)$ are cancelled to a large extent by the indirect effects u_0 of the forcing. Comparison of Fig.5 to Figs.3 and 4 shows that, in each pannel in Fig.5, the acceleration pattern in the upper troposphere is similar to that due to internal forcing $(F_1 + F_2)$ alone (Fig.3); whereas in the lower troposphere it is much more similar to that due to the separated external forcing F_3 (Fig.4). This is in good agreement with the well–known facts that generation of momentum is concentrated to the surface, and eddy momentum flux is much more developed in the upper troposphere. More profoundly, due to the different weighted contributions of internal and external forcing to the momentum variation in different parts of the atmosphere, the vertical shear of westerly changes prominently. In the cases of separated forc-

ing, the distributions of $[u]_t$ are relatively uniform in the vertical, with isolines vertically oriented as shown in Figs.3c and 4c. Whereas in the case of combination forcing, $[u]_t$ changes signs in the vertical, resulting in the declination of isolines. In mid–latitudes, acceleration in the upper troposphere due to internal forcing and deceleration in the lower troposphere due to external forcing cause an intensified vertically westerly shear. In tropical and subtropical latitudes, vertically easterly shear is basically maintained by upper westerly deceleration due to internal forcing and lower acceleration resulting from momentum generation. Therefore, the baroclinicity of the atmosphere measured in vertical westerly shear is to a great deal maintained dynamically by both internal and external mechanical forcing.

IV. THE INFLUENCES ON MEAN FLOW OF INTERNAL AND EXTERNAL THERMAL FORING

If there is no internal nor external mechanical forcing, $F_1 + F_2 = 0$, $F_3 = 0$. If these forcings are in balance, $F_1 + F_2 + F_3 = 0$. Then $u_t + u_s = 0$ in equation (47).Nevertheless, since either internal thermal forcing H_1 or / and H_2 or external forcing H_3 can excite secondary mean meridional circulation ψ (see equation (39)), and since the thermally induced mean meridional circulation can contribute to the westerly acceleration, thermal forcing can also affect the mean flow. In such circumstances, the tendency equation (47) reduces to

$$[u]_t = u_0(H_1, H_2, H_3).\tag{48}$$

Fig.6 shows the westerly tendency resulting from horizontal and vertical eddy heat transfer respectively. Horizontal eddy heat flux increases (decreases) westerly momentum in upper (lower) troposphere in low latitudes, whereas it decreases (increases) westerly momentum in upper (lower) troposphere in middle latitudes. As a result, vertically easterly shear in low latitudes and westerly shear in mid–latitudes are all reduced by internal thermal forcing H_1. Almost all main centres of $[u]_t$ are located in the lower stratosphere and at the surface, with intensities of about $1 \text{ms}^{-1}\text{d}^{-1}$, much weaker than their counterparts in the case of internal mechanical forcing (F_1). The effects of vertical eddy heat flux on the variation of $[u]$ are secondary (Fig.6b). Near the jet centre, the deceleration effects amount to 0.3 $\text{ms}^{-1}\text{d}^{-1}$ in the southern Hemisphere and 0.6 $\text{ms}^{-1}\text{d}^{-1}$ in the Northern Hemisphere. At about 600 hPa in each hemisphere right below the deceleration centre, there locates an acceleration maximum about half as strong as that aloft. In high latitudes, this forcing is much less important. The effect on the mean flow of the total thermal eddy–forcing $H_1 + H_2$ shown in Fig.7a thus resembles that due to H_1 alone.The effect of external forcing H_3 is shown in Fig.7b in comparison to that of internal forcing. This effect is more concentrated in low latitudes. In the tropics, in coordinate with the existence of double Hadley cells, there exist two layers of deceleration and acceleration structure. In subtropics, diabatic heating results in westerly deceleration. Maximum acceleration and deceleration are respectively found to be located at the tropopause and near the surface in the northern tropics with intensity of about 2 $\text{ms}^{-1}\text{d}^{-1}$.When both the internal and external thermal forcing are combined together, results shown in Fig.7c bear resemblance with those due to external forcing in low latitudes and those due to internal forcing in extratropics. Therefore, the total thermal effects on the mean flow in mid–latitudes are to reduce its vertical westerly shear against mechanical forcing. Comparison of Fig.7 with Fig.5 shows that the intensity of such thermal reduction effect on

mid—latitude westerly shear is only about one third of that of the mechanical production effect. We can therefore conclude that although the latitudinally differential diabatic heating might cause westerly shear in mid—latitudes as shown in Fig.7b, it is very weak. The total thermal effect is to reduce the mid—latitude baroclinity measured by the vertical shear of westerlies.

From results shown above, it is clear that mechanical forcing affects the mean flow much more strongly than thermal forcing does. Therefore, when thermal and mechanical forcing act together, the flow variation picture is very similar to that due to mechanical forcing alone. In Fig.8 are shown the distributions of total zonal momentum tendency resulting respectively from internal forcing ($F_1 + F_2 + H_1 + H_2$), external forcing ($F_3 + H_3$) and total forcing ($F+H$). These figures do not differ very much from their counterparts in the cases of pure mechanical forcing shown in Figs.3c, 4c and 5c respectively. Therefore, mechanical forcing takes the first place in the maintenance of zonal mean flow, and thermal forcing is secondary.

V. CONCLUSIONS AND DISCUSSIONS

Compared to the quasi—geostrophic theories, the non—acceleration theorem of the primitive equation system developed in Section II is more effective in understanding the wave—mean flow interactions against external forcing. This is more practical in that it not only includes the effects on the mean flow of vertical eddy transfer properties, but also takes full account of the impacts of external forcing and the forcing—induced mean meridional circulation. The latter processes are important when the intensity and structure of mean flow are investigated.

Mechanical forcing overwhelms thermal forcing in the acceleration of mean flow, in which the horizontal eddy flux of momentum plays the most important roles. The effects of internal mechanical forcing are mainly in the free atmosphere and concentrate near the tropopause. Mean westerlies are accelerated in mid—latitudes and decelerated in low and high latitudes. However, the effects of external forcing are more concentrated to the lower troposphere, causing the westerlies being decelerated in mid—latitudes and accelerated elsewhere. Therefore, total mechanical forcing is basical for the maintenance of vertically easterly shear in low latitudes and westerly shear in mid—latitudes.

Thermal forcing is not as important as mechanical forcing in contributing to the mean flow variations. External thermal forcing is important in low latitudes, whereas internal thermal forcing plays more important roles in extratropics. In mid—latitudes, the latter can reduce to a large extent the vertically westerly shear resulting from mechanical forcing.

Mean meridional circulation always counteracts either internal or external forcing so that the geostrophic and hydrostatic balances of the atmosphere can be maintained. However, its roles are passive and secondary for the variation of mean flow. The active and first roles are internal forcing or / and external forcing.

Since the internal mechanical forcing associated with horizontal eddy flux of momentum is dominant in the mean flow variations, as the first approximation, it is adequate to employ quasi—geostrophic system to study the interactions between waves and mean flow. However, since the mean meridional circulation in low latitudes in quasi—geostrophic system is too weak, and since the effects of mean meridional circulation are by no means negligible, when

the intensity and structure of the basic flow are concerned, primitive equation system would be more suitable.

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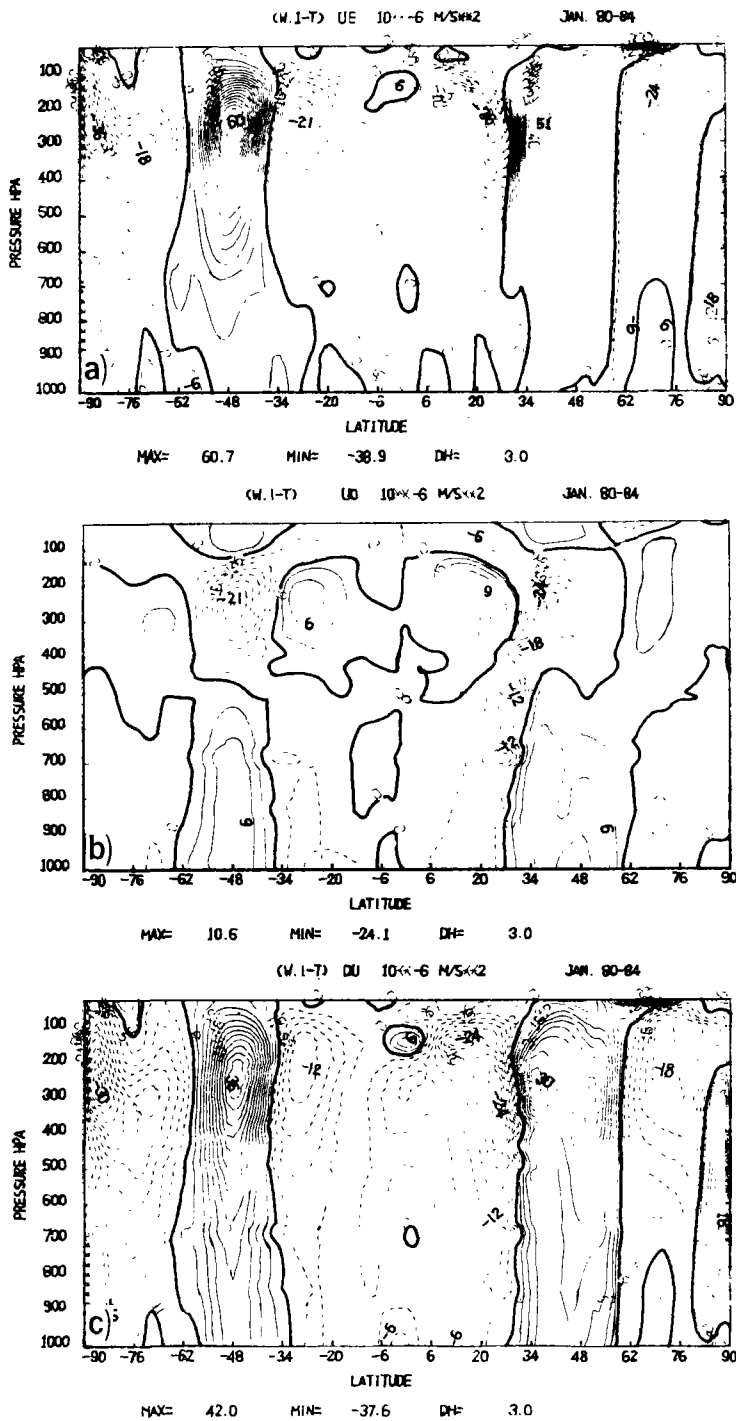
Fig.1. The temporal variation of zonal mean flow in January in connection with the internal forcing F_i of eddy horizontal flux of momentum. Intervals in $3 \times 10^{-6} ms^{-2}$. Refer to Eqs.(46) and (47) for detailed definitions. (a) u_L : due to direct eddy effect; (b) u_D : due to the indirect eddy effects; (c) $[u]_t$: momentum tendency.
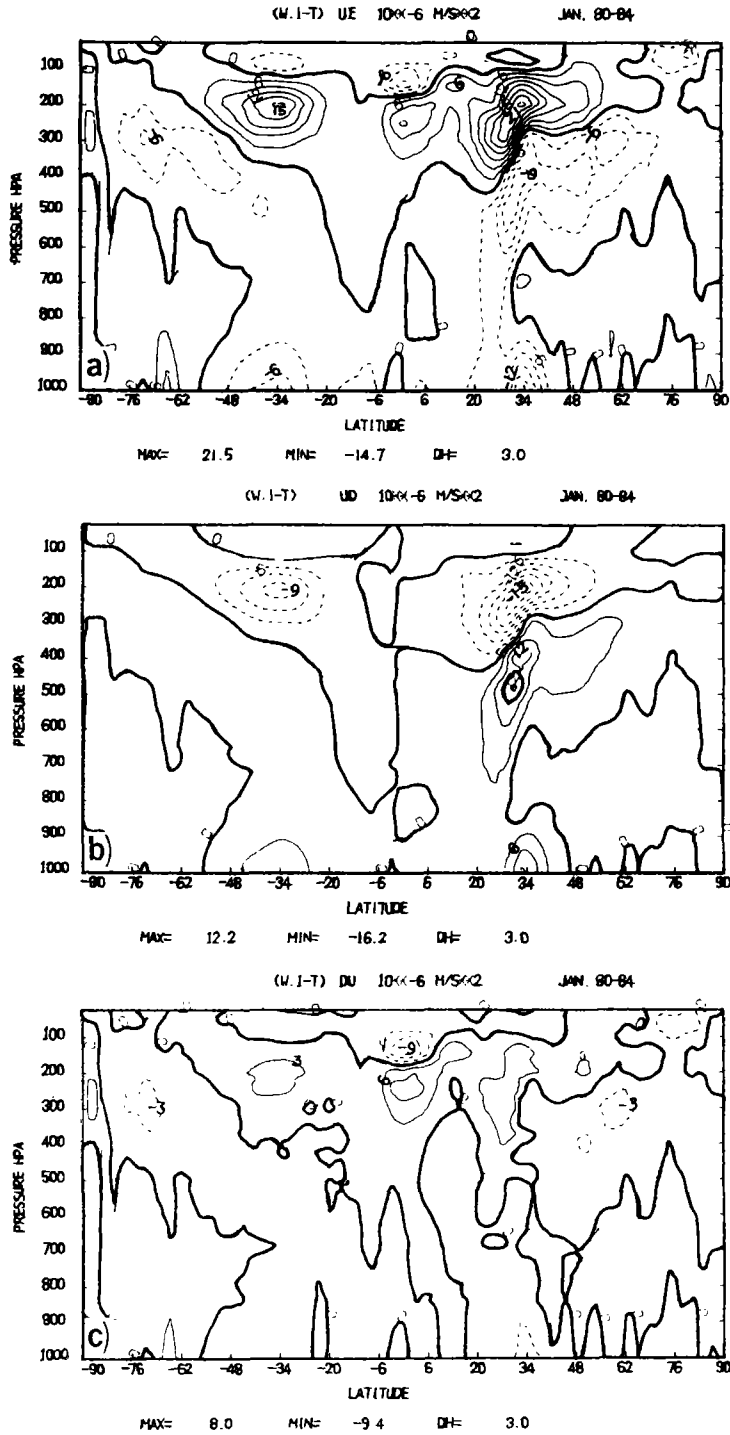
Fig.2. The temporal variation of zonal mean flow in January in connection with the internal forcing F_2 of eddy vertical flux of momentum. Intervals in 3×10^6 ms^{-2} .(see Fig.1 for legends).
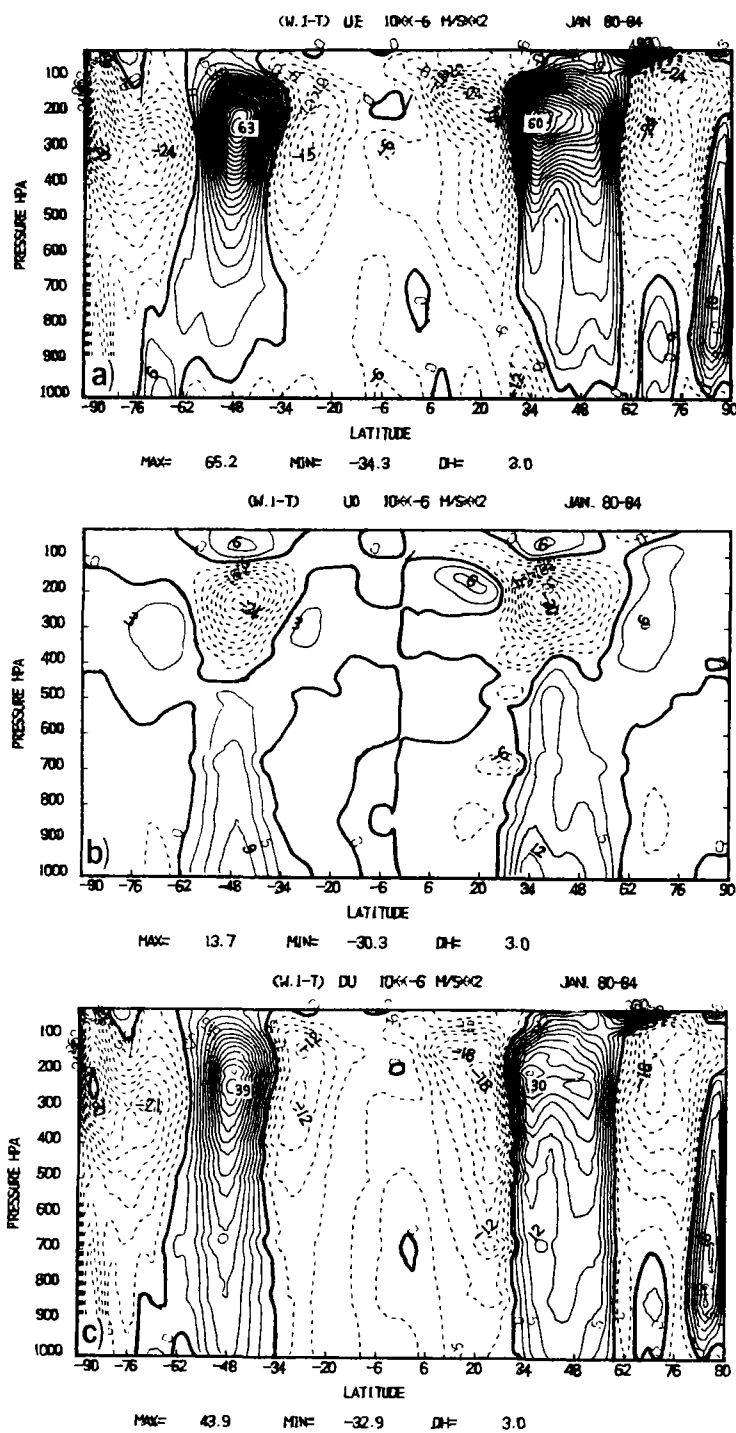
Fig.3. The temporal variation of zonal mean flow in January in connectionwith the total internal mechanical eddy forcing $F_1 + F_2$. Intervals in 3×10^{-6} ms^{-2} (See Fig.1 for legends).

(W. I-T)   US   10**-6   M/S**2   JAN. 80-84

MAX=   185.4   MIN= -210.5   DH=   4.5

(W. I-T)   UD   10**-6   M/S**2   JAN. 80-84

MAX=   109.0   MIN= -72.8   DH=   4.5

(W. I-T)   DU   10**-6   M/S**2   JAN. 80-84
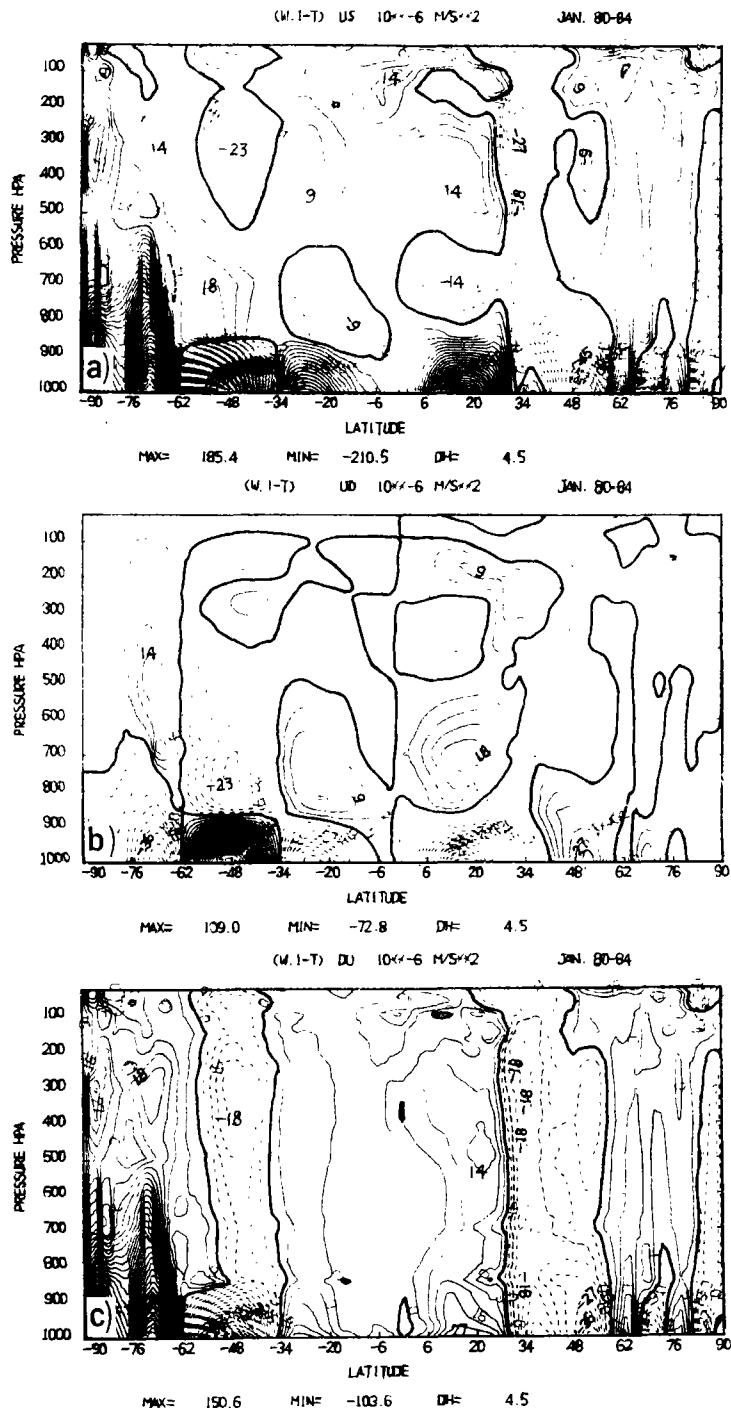
MAX=   150.6   MIN= -103.6   DH=   4.5

Fig.4. The temporal variation of zonal mean flow in January in connection with external mechanical forcing F_i Intervals in 4.5×10^{-6} ms^{-2} (See Fig.1 for legends).
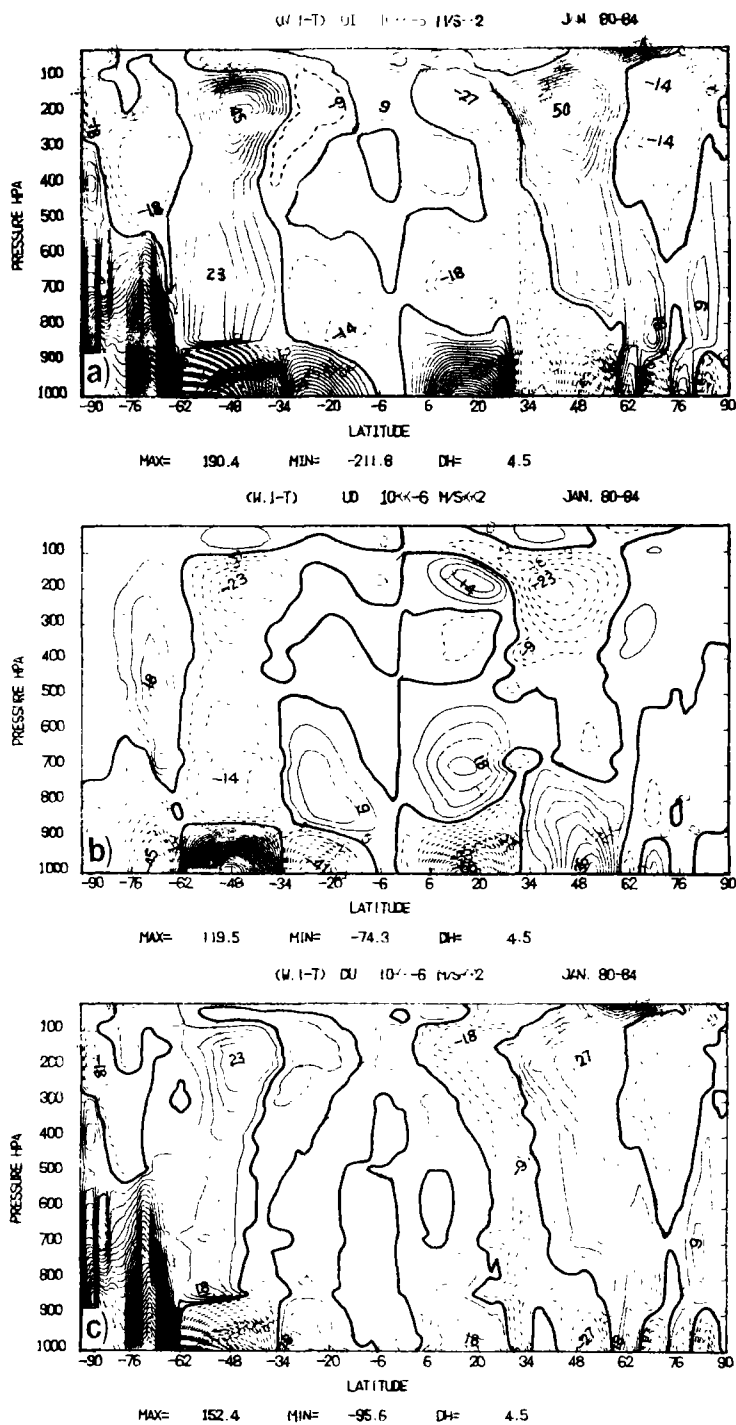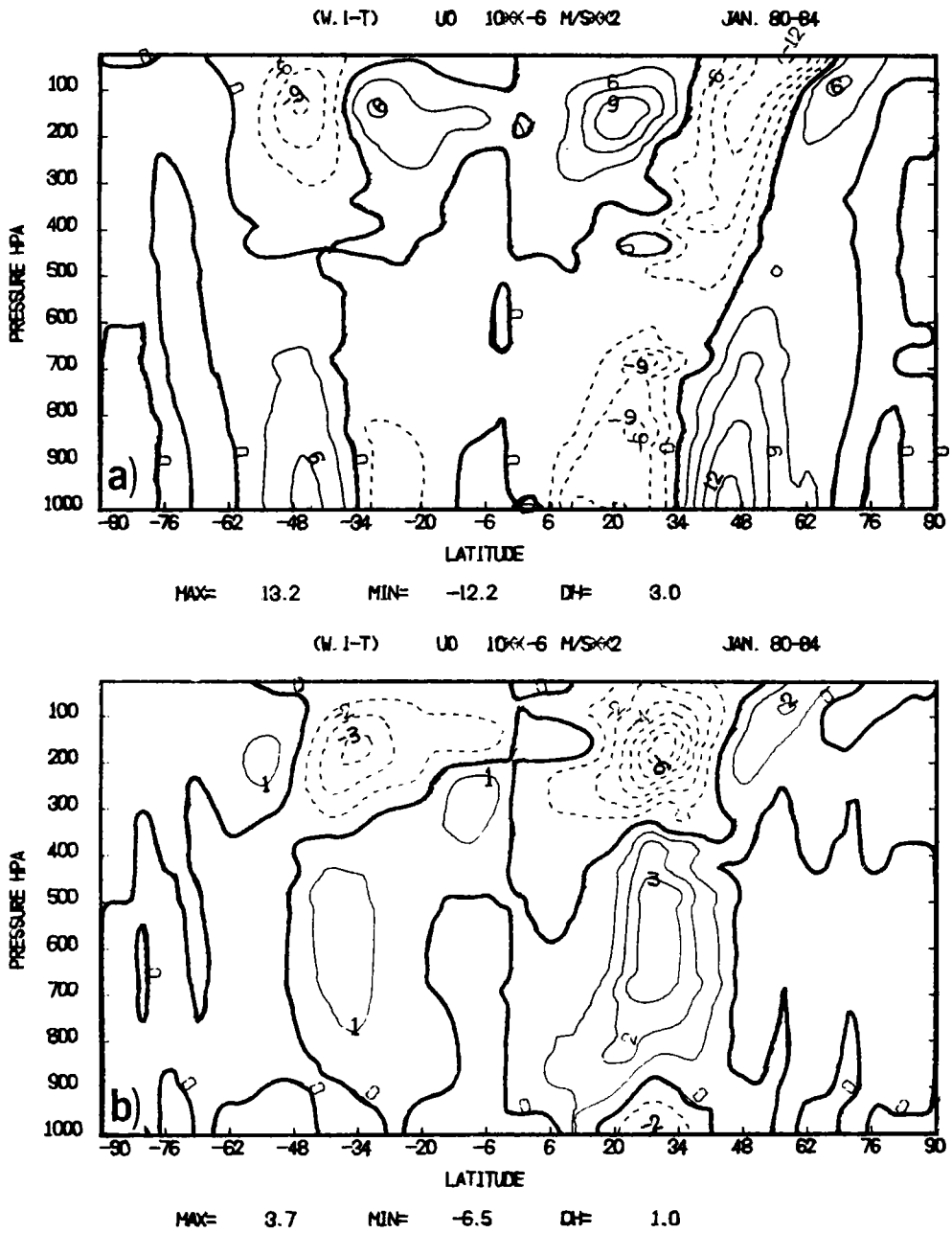
Fig.5. The temporal variation of zonal mean flow in January in connection with combined internal and external mechanical forcing f. Intervals in $4.5 \times 10^{-6} ms^{-2}$. (a) $u_f + u_s$: due to direct effects of mechanical forcing f; (b) u_0 due to indirect effects of mechanical forcing f; (c) $[u]_t$: total momentum tendency.

(W. I-T)   U0   10**-6 M/S**2   JAN. 80-84

MAX=   13.2   MIN=   -12.2   DH=   3.0

(W. I-T)   U0   10**-6 M/S**2   JAN. 80-84

MAX=   3.7   MIN=   -6.5   DH=   1.0

Fig.6. The temporal variation of zonal mean flow in January in connection with internal thermal forcing. Units in 10^{-6} ms^{-2}. (a) momentum tendency $[u]_t$ due to forcing H_1 ; (b) momentum tendency $[u]_t$ due to forcing H_2.

(W.1-T)   UO   10XX-6 M/SXX2   JAN. 80-84

MAX=   13.5   MIN=   -11.5   DH=   3.0

(W.1-T)   UO   10XX-6 M/SXX2   JAN. 80-84

MAX=   20.2   MIN=   -22.4   DH=   3.0

(W.1-T)   UO   10XX-6 M/SXX2   JAN. 80-84
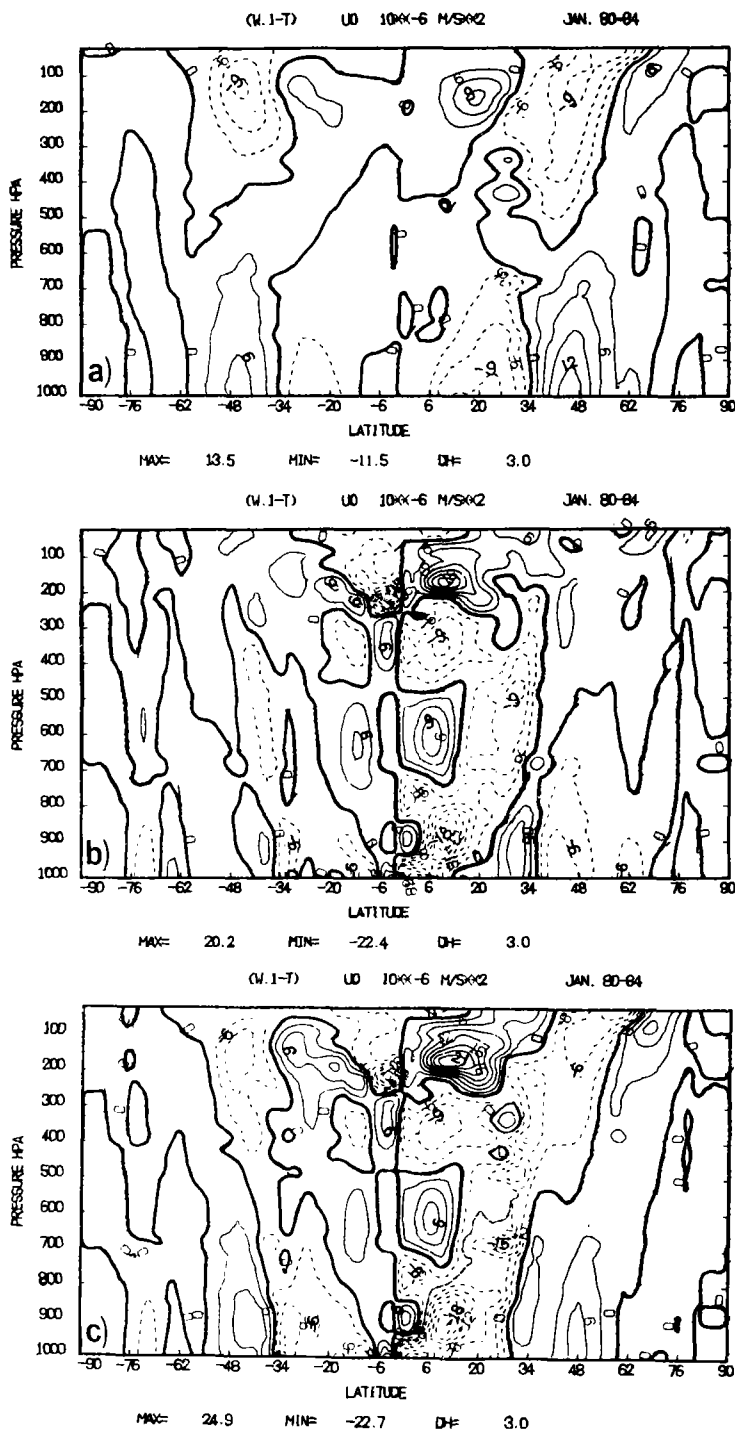
MAX=   24.9   MIN=   -22.7   DH=   3.0

Fig.7. The temporal variation of zonal mean flow in January in connection with thermal forcing. Intervals in 3 $\times 10^{-6}$ ms^{-2}. (a) due to internal forcing $H_1 + H_2$; (b) due to external forcing H_3; (c) due to total thermal forcing H.
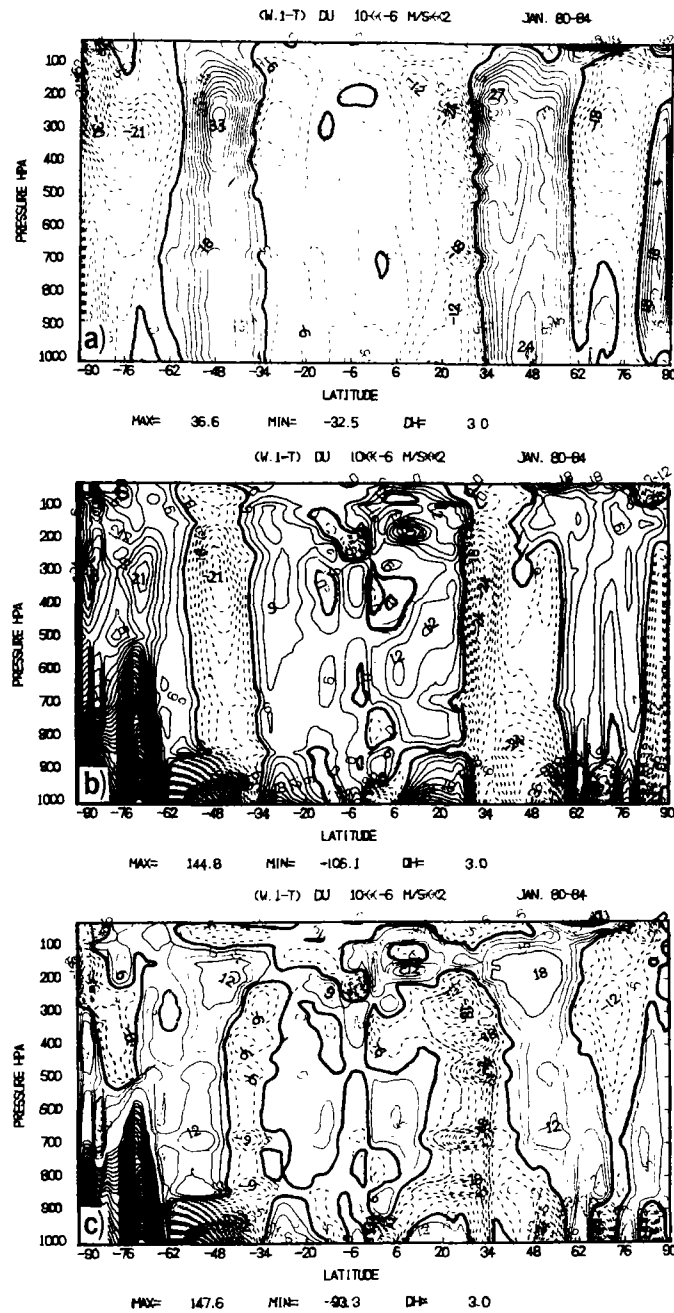
Fig.8. The temporal variation of zonal mean flow in January in connection with different categories of combined thermal and mechanical forcing. Intervals in 3×10^{-6} ms^{-2}. (a) $[u]_t$ due to total internal forcing $F_1 + F_2 + H_1 + H_2$; (b) $[u]_t$ due to total external forcing $F_3 + H_3$; (c) $[u]_t$ due to total internal and external forcing $F + H$.